Delegated Portfolio Management with Socially Responsible Investment Constraints

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Abstract

We consider the problem of how to set a compensation for a portfolio manager who is required to restrict the investment set, as it happens when applying socially responsible screening. This is a problem of Delegated Portfolio Management where the reduction of the investment opportunities to the subset of sustainable assets involves a loss in the expected earnings for the portfolio manager, compensated by the investor through an extra bonus on the realized return. Under simple assumptions on the investor, the manager and the market, we compute the optimal bonus as a function of the manager's risk aversion and his expertise, and of the impact of the portfolio restriction on the Mean Variance efficient frontier. We conclude by discussing the problem of selecting the best managers when his ability is not directly observable by the investor.

Keywords: Delegated Portfolio Management; Socially responsible investment; Incentives; Extrinsic incentives and intrinsic motivations.

1 Introduction

Socially Responsible Investment (SRI), that is the selection of assets of firms satisfying some socially responsible criteria, is receiving a growing attention from private and institutional investors. Although mainstream Economic theory consider Social Responsibility as a non-financial issue that most likely yields extra costs on the firm, there are many studies that question such an approach and recognize a positive value, also in terms of financial performances, to SRI. Mill (2006), like many other studies, try to shed some light on this point, examining the financial performance of a socially responsible investment over time. Platinga and Scholtens (2001), Stone et al. (2001) and Bauer et al.(2007) compare the performances of SRI funds to those of conventional funds. Kempf and Osthoff (2007), separate the performances of the portfolios from the skill of fund managers, compared specially constructed SRI and non-SRI portfolios. The general picture is still not clear and the evidence is mixed, because of different data sets and different interpretation, however a consistent group of researcher and of investors, believes in the possibility of "Doing well while doing good". On the other hand, it emerges for instance by the report by RImetrics (2008), that portfolio managers appear to be reluctant to include SRI into their investment strategies, mostly for the obvious considerations that they are worried that a restriction of their investment set may be costly in terms of generated returns and, as a consequence on their bonuses. The role of portfolio managers is key in mainstreaming SRI polices, hence it is important to study the problem of how to encourage them to move in such a direction. Sundblad et al (2009) present an analysis on their attitudes toward policy regulations, we are interested here in a theoretical study on how to set correct incentives.

We consider the problem of an investor who wants to invest her wealth according to a socially responsible rule and to this end delegates a portfolio manager, who has a superior knowledge of the market. The manager is paid a fee that is proportional to the total return produced by the portfolio. The market is composed of "green"and "non-green "assets[1].

[1]Hereafter we call "green"those assets that fulfil a given SRI requirements and we call "non-green "all others assets. Consequently we call "green"those investors who trade only in green assets, and "conven-

A socially responsible investor wants the manager to invest only in "green" assets. Such a constraint reduces the investment opportunities of the manager and has therefore a negative impact on his expected compensation, hence the investor compensates him by increasing the bonus. How is such a bonus related to the characteristics of the market, of the agents, and of the skill of the agents? Those are the kind of questions that we address in the paper.

The problem that we formulated is a stylized version of a rather common situation when an institutional investor wants to invest by maximizing the expected return, but also obeying to some non-financial constraint. The investor may also be interpreted as a government who sets a fiscal bonus to encourage some investments in spite of others.

Our approach to the problem belongs to the set of the Delegated Portfolio Management (DPM) literature, see Stracca (2005) for a review. The classical problem of DPM is that of designing an appropriate contract when the managers information and the effort expended are not directly observable by the investors. Consequently the appropriate contract should motivate the manager to exert costly effort to gather information and also induce the portfolio manager to subsequently use such information in choosing a portfolio with desirable risk characteristics. The literature focuses principally on optimal contract functions and their effects like in Stoughton (1993), Wei and LiTiwari(2009) and Admati and Pfleiderer (1997), but, rather surprisingly, it has devoted little attention to the restriction commonly found in the investment policies that defines the contracts between investors and managers, as shown by Almazan et al (2004). One notable exception is Gomez and Sharma (2006) where one of the most common constraints, short selling, is taken into account.

We do not address the problem of which contract function is preferable in case of SRI constraints to avoid moral hazard or to answer others of the usual questions of DPM. The main objective of our study is to formulate a model that is simple enough to get explicit results but also sufficiently structured to address some important issues such as the impacts of the manager's risk aversion, of his skill and of the loss in diversification opportunities,

tional "those who trade any asset

on the compensation scheme when a constrained investment mandate is proposed. To this goal we consider a single-period model where the allocation universe consists of a number of risky assets, only some of which can be considered as "green", according, for instance, to one, or all, of the common screening policies considering environmental, social and governance issues, and the rest are "non-green ". The market also contains a riskless asset that is considered as green. Of course, restricting the investment set reduces the expected utility and yields to what can be called a "sustainability cost ". Whereas the common approach is to consider a reservation utility that is exogenously determined, here we relate it to a possible alternative conventional contract for the manager. To compensate for the reduction in the set of investment opportunities, the socially responsible investor offers an extra fee on the return of his allocation strategy with respect to a conventional fund, that we call the "green bonus ". By increasing the incentive fee, the investor expands the manager's opportunities, thereby partially undoing the effects imposed by SRI screening. Therefore, the problem for the green investor is that of setting the bonus and select a competent manager. Indeed, as in Bhattachary and Pfleiderer (1985), we assume that, after being employed, the manager observes a private signal that can be used to forecast the future returns of assets. The investor's goal is to design a contract that is not too expensive for her but sufficient to attract a manager with a sufficient skill. Bhattachary and Pfleiderer (1985) consider a market with a single risky asset and hence the efficiency of the manager, defined as the precision of his forecast, could be measured by the variance of the private signal (a positive real number). In our case, the market is composed by many assets and hence the variance of the signal perceived by the manager is a positive definite matrix. This arises some new issues on the ranking of the managers based on their skills.

Adopting a linear sharing rule, assuming a risk-neutral investor and a risk-averse portfolio manager and considering normal returns, we derive an explicit formula for the green bonus that takes into account the manager's expertise and relates the bonus to the statistical features of the assets involved and/or discarded by the screening. Our setting allows us to decompose the bonus into two terms, one taking into account the missed investment and risk diversification opportunities, the other one compensating the missed exploitation of the signal by the manager, that is the fact that the manager receives a signal on the whole market but he can exploit it only on the green assets. We study the properties of such a bonus and propose an example applying the formula to market data in the case of screening of financial assets from S&P500 of December 2006 according to some SRI criteria. Finally, considering the managers' selection problem, we show that managers with higher expertise on the green assets require a lower bonus than equally skilled, but less "green-focused" managers, that is, as in Kreps (1997), stronger intrinsic motivations require smaller extrinsic incentives.

The rest of the paper is organized as following: in Section 2 we present the model and the optimisation problem; in Section 3 we derive the solution in two cases, with and without private signal; discussing the result about the green bonus and introducing a definition of manager's expertise; finally we discuss the problem of manager's selection.

2 The model

We consider a one period economy, with two agents: a risk neutral investor, and a portfolio manager with exponential utility function $u(x) = -\exp(-\alpha x)$, where the coefficient of risk aversion α is common knowledge. There are n risky assets with return $\mathbf{X}$ distributed as a multivariate normal with mean $\bar{\mathbf{X}}$ and variance Σ and one riskless asset with return R. The risky investment set can be partitioned into two disjoint subsets, the first one composed by p "green" assets with return $\mathbf{P}$, normally distributed with mean $\bar{\mathbf{P}}$ and variance Σ_P and by q "non-green "assets $\mathbf{Q}$, normally distributed with mean $\bar{\mathbf{Q}}$ and variance Σ_Q. In other terms we have

$$\mathbf{X} = \begin{pmatrix} \mathbf{P} \\ \mathbf{Q} \end{pmatrix}, \quad \bar{\mathbf{X}} = \begin{pmatrix} \bar{\mathbf{P}} \\ \bar{\mathbf{Q}} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_P & \Sigma_{PQ} \\ \Sigma_{PQ}^T & \Sigma_Q \end{pmatrix};$$

where Σ_{PQ} is an $p \times q$ matrix and the superscript T represents transposition. Our setting is an extension of Bhattachary and Pfleiderer (1985) to the case of n risky assets and the distinction between "green" and "non-green" assets.

The manager receives a noisy signal on the return $\mathbf{X}$, given by

$$\mathbf{S} = \mathbf{X} + \epsilon,$$

where $\mathbf{X}$ and ϵ are uncorrelated and ϵ is a normal n-dimensional random variable with mean zero and variance Σ_ϵ. The random vector ϵ is the noise of the signal, hence the skill of a manager is represented by the variance Σ_ϵ. The manager decides on the investment after having observed a value S.

Following Bhattachary and Pfleiderer (1985) we can show that the conditional distribution of $\mathbf{X}$ given $\mathbf{S} = S$ is a normal random variable with mean

$$\mathbf{M}(S) = \bar{\mathbf{X}} + \Sigma\Sigma_S^{-1}(S - \bar{\mathbf{X}})$$

where $\Sigma_S = \Sigma + \Sigma_\epsilon$ and variance

$$V = \Sigma - \Sigma\Sigma_S^{-1}\Sigma.$$

Note that V is a symmetric positive definite matrix that does not depend on S.

To distinguish between green and non-green assets components we define

$$\mathbf{M}(S) = \begin{pmatrix} \mathbf{M_P}(S) \\ \mathbf{M_Q}(S) \end{pmatrix}, \quad V = \begin{pmatrix} V_P & V_{PQ} \\ V_{PQ}^T & V_Q \end{pmatrix}.$$

The SRI investor allocates a capital W_0 to a manager with the mandate to trade only on the green assets and on the riskless one. The compensation of the manager is

$$f(W) := AR + bW, \tag{1}$$

where the parameter A is a fixed amount received at the beginning of the period and b is the fee on the realized wealth, that is given by

$$W(\omega_P) = \omega_P^T \mathbf{P} + \left[W_0 - \mathbf{1}_p^T \omega_P\right] R, \tag{2}$$

where ω_P is a p dimensional vector representing the allocation in the green assets and $\mathbf{1}_p$ is the p dimensional vector of all ones. We assume that the manager can also choose a second contract without investment restriction, whose payoff is

$$r(W') := AR + b_0 W', \tag{3}$$

and where

$$W'(\omega) = \omega^T \mathbf{X} + \left[W_0 - \mathbf{1}_n^T \omega\right] R, \tag{4}$$

where ω is an n dimensional vector representing the allocation in the whole investment set (green and non-green assets). The difference:

$$\Delta := b - b_0$$

between the two fees is the "green bonus" and reflects the compensation for the restriction on the investment set.

The manager observes the signal only after having accepted the contract. After it, he determines the optimal allocation ω_P^* by optimising his expected utility

$$\mathbb{E}\left[\ u\left(f(W(\omega_P))\right) | \mathbf{S} = S\ \right] \tag{5}$$

allocating the wealth W_0 in the green assets. The decision on whether or not accepting the contract must be taken before observing the signal on the basis of the expected utility over all possible signals. Of course, the decision depends on the bonus Δ offered by the investor. Hence the manager accepts the green contract if the participation constraint

$$\mathbb{E}\ [\ u(f(W))\ ]\ \geq\ \mathbb{E}\ [\ u(r(W'))\ ]; \tag{6}$$

is satisfied. We refer to the right hand side of (6) as the manager's "reservation utility ".

Note that both W and W' depend on the manager's allocation ω_P and ω, respectively, which depend on the observed signal S and also on the matrix Σ_ϵ, moreover the optimal allocation ω_P^* (resp. ω^*) is a function of the contract parameter b (resp. b_0), thus ω_P^* is a function of Δ. In the following, we explicitly indicate only one or two of all these dependencies at a time for the sake of a simpler notation.

The principal maximises the expectation of her final wealth after rewarding the manager. We remark that we do not impose any short-sale constraint and that the compensation of the manager may also turn out to be negative, that is the contract does not have a limited liability. Putting all pieces together, we state the optimisation problem faced by the investor

$$\max_{\Delta} \mathbb{E}\ \left[\ W(\omega_P^*(\Delta, \mathbf{S})) - f(W(\omega_P^*(\Delta, \mathbf{S})))\ \right] \tag{7}$$

$$\omega_P^*(\Delta, S) = \arg\max_{\omega_P} \mathbb{E}\ \left[\ u\left(AR + (b_0 + \Delta)W(\omega_P)\right) | \mathbf{S} = S\ \right] \tag{8}$$

$$\mathbb{E}\ [\ u(f(W))\ ]\ \geq\ \mathbb{E}\ [\ u(r(W'))\ ] \tag{9}$$

Note that, to not overload formulas, we did not report explicitly the dependence on all the input variables. In particular, we wish to remind that the solution of the problem depends on the manager's skill Σ_ϵ.

3 The Optimal Bonus

In the following section we solve the problem specified in (7),(8),(9). First we solve it in the simpler case where the manager does not receive any private signal. Later we solve the general case with private information. The separate study of these two cases allows us to decompose the green bonus into two terms, the first one taking into account the differences in the investment opportunities, the second one considering the potentiality of the unexploited signal in virtue of the investment restriction. Thus we can distinguish how much of the bonus depends on the properties of the market and how much on the manager's expertise. This will be useful to set the bonus to select a manager of a sufficient expertise.

3.1 The case of managers without private information

We present here a simplified version of the model where the investor and the manager have access to the same information on the asset's returns and the investor needs to hire the manager because she cannot directly trade. This situation is equivalent to the case where the signal is so noisy that the observation of $\mathbf{S}$ does not provide any valuable information to the manager.

The investor's problem in this case is given by

$$\max_{\Delta} \mathbb{E} \; [\; W(\omega_P^*(\Delta)) - f(W(\omega_P^*(\Delta))) \;] \tag{10}$$

$$\omega_P^*(\Delta) = \arg\max_{\omega_P} \mathbb{E} \; [\; u\left(AR + (b_0 + \Delta) W(\omega_P)\right) \;] \tag{11}$$

$$\mathbb{E} \; [\; u(f(W)) \;] \geq \mathbb{E} \; [\; u(r(W')) \;] \tag{12}$$

whose solution is in the following Proposition.

Proposition 1 *The solution of the problem (10) (11) (12) is*

$$\bar{\Delta}_0 = \frac{\mathcal{H} - \mathcal{H}_P}{2\alpha W_0 R}, \tag{13}$$

where $\mathcal{H}_P = (\bar{\mathbf{P}} - R\mathbf{1}_p)^T \Sigma_P^{-1} (\bar{\mathbf{P}} - R\mathbf{1}_p)$ *and* $\mathcal{H} = (\bar{\mathbf{X}} - R\mathbf{1}_n)^T \Sigma^{-1} (\bar{\mathbf{X}} - R\mathbf{1}_n)$*.*

Proof. We prove first that the manager participation constraint (12) is satisfied if $\Delta \geq \bar{\Delta}_0$ and then we show that the principal expected utility is decreasing respect to Δ.

The expected utility on the left hand side of (12) is given by

$$\mathbb{E}\left[-e^{-\alpha f(W)}\right] = -e^{-\alpha AR} \cdot e^{-\alpha b \mu_W + \frac{(\alpha b)^2}{2}\sigma_W^2} \tag{14}$$

since W is normally distributed.

The solution of (11) is

$$\omega_P^* = \frac{1}{\alpha(b_0 + \Delta)}(\bar{\mathbf{P}} - R\mathbf{1}_p)\Sigma_P^{-1}.$$

Hence the maximum expected utility obtained substituting ω_P^* into (14) is

$$\mathbb{U}_0(f(W)) := \mathbb{E}\left[-e^{-\alpha f(W(\omega_P^*(\Delta)))}\right] = -e^{-\alpha AR} \cdot e^{-\frac{\mathcal{H}_P}{2}} \cdot e^{-\alpha(b_0+\Delta)W_0 R}. \tag{15}$$

Analogously the reservation utility on the right hand side of (12), is given by

$$\mathbb{U}_0(r(W')) := \mathbb{E}\left[-e^{-\alpha r(W')}\right] = -e^{-\alpha AR} \cdot e^{-\frac{\mathcal{H}}{2}} \cdot e^{-\alpha b_0 W_0 R}. \tag{16}$$

Substituting into (12) we get (13).

The expected value of the principal's wealth, obtained by substituting the optimal solution ω_P^* into (10), is

$$E[W_P] = (1 - (b_0 + \Delta))\left[W_0 R + \frac{\mathcal{H}_P}{\alpha(b_0 + \Delta)}\right] - AR$$

that is a decreasing function of Δ, hence the principal chooses the smallest Δ satisfying the constraint. □

The value $\bar{\Delta}_0$ is the minimal bonus necessary to convince an agent (who does not receive any private signal) to accept the "green" mandate. Note that $\mathcal{H}$ and $\mathcal{H}_P$ are the squared Sharpe Ratios of the total market portfolio and of the "green" market, respectively, hence $\bar{\Delta}_0$ is always positive. The Sharpe Ratios' difference measures the loss of expected risk adjusted returns. Figure 1 gives an interpretation of the effect of the green bonus.

The left panel shows the green efficient frontier (dashed line) that of course is below the conventional one (solid line). Recall that the slopes are given by the Sharpe ratios. The two optimal allocations (green and conventional) belong to two different utility curves. In the right panel it can be seen how the green bonus $\bar{\Delta}_0$ shifts the green efficient frontier upwards (dash-dot line) so that the new green optimal allocation now belongs to the same utility curve as the conventional one.

The optimal green bonus $\bar{\Delta}_0$ is inversely proportional to the risk aversion and to W_0 that represents the Asset Under Management. The inverse relation with the risk aversion is explained by the fact that more risk averse managers invest less in the risky assets and more in the risk-free asset, hence the loss in diversification is smaller and, consequently, a smaller bonus is required. Figure 2 represents $\bar{\Delta}_0$ as a function of relative risk aversion $\frac{\alpha}{\alpha_M}$ for three different screening criteria, where α_M is the risk aversion of the investor who chooses the market portfolio. Input data for this example are the monthly return of the S&P index assets, December 2006, screened via ESG criteria using strengths and concerns provided by KLD. The screening process excludes 147, 336 and 417 assets (on 488) respectively for the Environment, the Governance and the Social dimensions. Obviously, a stronger screening implies a higher green bonus. The case considered shows that the green bonus has to be rather small (never higher than 20 basis points), because the loss in terms of Sharpe Ratio due to the screening is in all these examples very small.

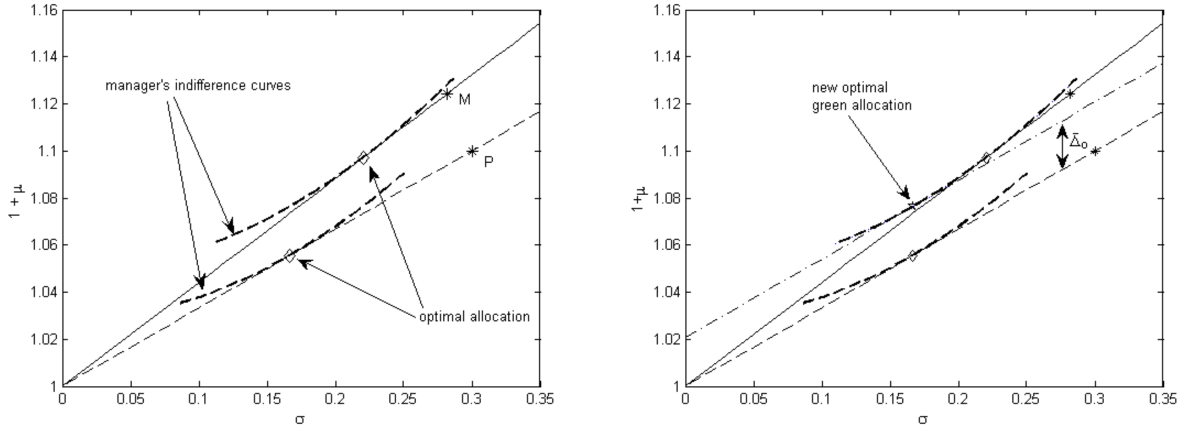

Figure 1: The Figure represents the effect of the green bonus on the manager's allocation and utility. The solid line and the dashed line are, respectively, the conventional and the green efficient frontiers. The left panel shows the conventional optimal allocation and the green one without a bonus. Of course, the green allocation provides the manager a lower utility than the conventional one. The right panel shows that the minimum green bonus $\bar{\Delta}_0$ shifts the green efficient frontier upwards (dashdot line) so that the green optimal allocation gives the same utility of the optimal conventional allocation

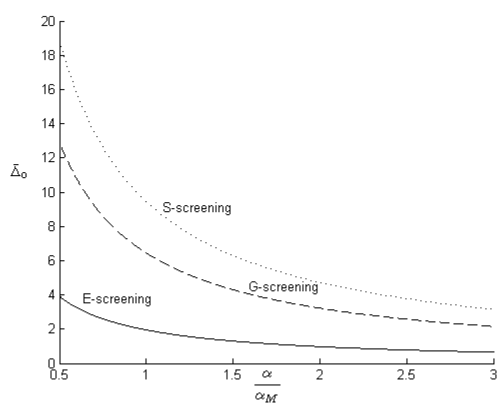

Figure 2: The minimum green bonus $\bar{\Delta}_0$ (in basis points) is plotted in function of α/α_M (risk aversion divided market risk aversion) for three different screening. The screening process has excluded the 30% , 69%, 85% of assets respectively for the Environment, the Governance and the Social dimensions.

3.2 The case of private information

Now we consider the case of managers that receive a private signal with a variance Σ_ϵ. In this case, the optimal bonus is given by the following

Proposition 2 *The solution of the optimisation problem (7), (8), (9) is*

$$\bar{\Delta} = \bar{\Delta}_0 + \Phi, \tag{17}$$

where $\bar{\Delta}_0$ *is given by (13) and*

$$\Phi = \frac{1}{2\alpha W_0 R} \log\left(\frac{\det(V^{-1}\Sigma)}{\det(V_P^{-1}\Sigma_P)}\right). \tag{18}$$

Proof. First we consider the manager's participation constraint (6) then we show that the principal utility is decreasing in Δ, and hence the thesis follows.

The optimal allocations after observing the signal S are

$$\omega_P^* = \frac{1}{\alpha(b_0 + \Delta)} V_P^{-1}(\mathbf{M}_P(S) - R\mathbf{1}_p) \tag{19}$$

in the case of the green contract and

$$\omega^* = \frac{1}{\alpha b_0} V^{-1}(\mathbf{M}(S) - R\mathbf{1}_n), \tag{20}$$

otherwise. The corresponding expected utilities are

$$\mathbb{E}\left[\ u(f(W))|\mathbf{S} = S\ \right] = -e^{-\alpha AR} \cdot e^{-\alpha(b_0+\Delta)W_0 R} \cdot e^{-\frac{\mathcal{H}_P(S)}{2}} \tag{21}$$

and

$$\mathbb{E}\left[\ u(r(W'))|\mathbf{S} = S\ \right] = -e^{-\alpha AR} \cdot e^{-\alpha b_0 W_0 R} \cdot e^{-\frac{\mathcal{H}(S)}{2}}, \tag{22}$$

where

$$\mathcal{H}(S) = (\mathbf{M}(S) - R\mathbf{1}_n)^T V^{-1}(\mathbf{M}(S) - R\mathbf{1}_n)$$

and

$$\mathcal{H}_P(S) = (\mathbf{M}_P(S) - R\mathbf{1}_p)^T V_P^{-1}(\mathbf{M}_P(S) - R\mathbf{1}_p).$$

To compute the expectations of (21) and (22) we consider the random variables $\mathbf{M}(\mathbf{S}) \sim N(\bar{\mathbf{X}}, Q)$ and $\mathbf{M}_P(\mathbf{S}) \sim N(\bar{\mathbf{P}}, Q_P)$, where $Q = \Sigma\Sigma_S^{-1}\Sigma$ and Q_P is the sub-matrix of Q composed by its first p rows and p columns. Let us define

$$\varphi(A, B, \nu) := \mathbb{E}\left[e^{-\frac{v^T A v}{2}}\right],$$

where A is a $n \times n$ positive definite matrix and v is a n-dimensional normal random variable with mean ν and variance B. A standard computation gives

$$\varphi(A, B, \nu) = \frac{e^{-\frac{c}{2}}}{\sqrt{\det(AB + I_n)}}, \tag{23}$$

with $c = \nu^T(B^{-1} - B^{-1}H^{-1}B^{-1})\nu$, $H = (A + B^{-1})$ and I_n the identity matrix of dimension n. Since $\mathbf{M}(\mathbf{S})$ and $\mathbf{M}_P(\mathbf{S})$ are normally distributed,

$$\mathbb{E}\ [\ \mathbb{E}\ (u(f(W))|\mathbf{S})\ ] = -e^{-\alpha AR} \cdot e^{-\alpha(b_0+\Delta)W_0R} \cdot \varphi(V_P^{-1}, Q_P, \mu_P)$$

and

$$\mathbb{E}\ [\ \mathbb{E}\ (u(r(W'))|\mathbf{S})\ ] = -e^{-\alpha AR} \cdot e^{-\alpha b_0 W_0 R} \cdot \varphi(V^{-1}, Q, \mu)$$

where $\mu = \bar{\mathbf{X}} - R$, $\mu_P = \bar{\mathbf{P}} - R$, hence we have

$$\bar{\Delta} = \frac{1}{\alpha W_0 R} \log\left(\frac{\varphi(V_P^{-1}, Q_P, \mu_P)}{\varphi(V^{-1}, Q, \mu)}\right) \tag{24}$$

Then we obtain that manager's participation constraint is satisfied for $\Delta \geq \bar{\Delta}$, with $\bar{\Delta}$ as in (17), by trivial substitution after observing that the expected utility is an increasing function of Δ. The principal's expected wealth is

$$E[W_P] = (1 - (b_0 + \Delta))\left[W_0 R + \frac{E[\mathcal{H}_P(\mathbf{S})]}{\alpha(b_0 + \Delta)}\right] - AR$$

that is decreasing with respect to Δ, hence we have the thesis. $\square$

Note that Φ is positive since $\mathcal{H}_P(S) \leq \mathcal{H}(S)$ for any S and hence $E[e^{-\mathcal{H}_P(\mathbf{S})}] \geq E[e^{-\mathcal{H}(\mathbf{S})}]$. Therefore $\varphi(V_P^{-1}, Q_P, \mu_P) \geq \varphi(V^{-1}, Q, \mu)$ and

$$\frac{\sqrt{\det(V^{-1}\Sigma)}}{\sqrt{\det(V_P^{-1}\Sigma_P)}} \geq \frac{e^{\frac{\mu_P^T \Sigma_P^{-1} \mu_P}{2}}}{e^{\frac{\mu^T \Sigma^{-1} \mu}{2}}}$$

for any μ. Since the right hand side is equal to one when $\mu = 0$, the left hand side is always greater than one.

While the term $\bar{\Delta}_0$ measures the cost of sustanaibility in terms of diversification and earnings opportunity, Φ reflects the opportunity cost of unexploited information.

4 Manager's expertise and selection

Now we propose a definition for manager's expertise in relation to the matrix which represents the skills and we discuss the problem of manager's selection. We call total expertise of a manager the quantity:

$$H := \frac{\det(\Sigma)}{\det(V)}.$$

Not that H is always greater than one. Analogously, we call $H^g = \det(V_P^{-1}\Sigma_P)$ the green expertise and $H^c = \det(V_Q^{-1}\Sigma_Q)$ the non-green expertise. We note that two managers with different skills (represented by Σ_ϵ) may have the same total expertise H, as it may be the case for two manager with a comparable level of expertise on different sectors. The term Φ is proportional to the ratio between the total and the green expertises, hence it depends on the relative knowledge of the green market held by the manager.

$$\Phi = \frac{1}{2\alpha W_0 R} \log\left(\frac{H}{H^g}\right). \tag{25}$$

Note that Φ is increasing with respect to the total expertise H and decreasing with respect the green expertise H^g. This means that if two managers have the same total expertise H, the one with higher expertise on the green assets requires a lower bonus. In the extreme case of $H^g = H$, when the agent's expertise is concentrated in the green assets, no efficiency bonus is required.

The term Φ measures the opportunity cost related to the unexploited information due to the investment restriction. In fact the manager receives a signal on all the available assets, but he can exploit and traduce into earnings only those information related to the green assets which include not only information regarding directly the green but any other information on the non-green ones which can be exploited through the correlation. A manager whose expertise is concentrated in the green assets is not losing any opportunity when asked to restrict the investment universe, and hence does not require any extra compensation. In fact if the correlation between two assets is strong a signal on one of them is useful also for estimating the other. Consider a Σ_ϵ diagonal, it can be show in case $n = 2$ by a straightforward computation that Φ reaches a maximum when ρ is equal zero.

Let us consider the case of two assets (one green and one conventional) with correlation ρ. Figure 3 shows the green bonus $\bar{\Delta}$ with the two components $\bar{\Delta}_0$ and Φ as a function of ρ. The term $\bar{\Delta}_0$ grows unbounded as ρ approaches 1 or -1 because when the assets are strongly correlated the diversification opportunity is so high that the manager would not give up it; the minimum is reached for $\rho = \frac{\mathcal{H}_P}{\mathcal{H}_Q}$, if $\mathcal{H}_P < \mathcal{H}_Q$ (otherwise for $\rho = \frac{\mathcal{H}_Q}{\mathcal{H}_P}$). On the contrary the minimal Φ is reached when ρ is 1 or -1, because when the assets are very correlated, the signal gives information with the same accuracy over all the assets (green and non-green) hence the manager can exploit it completely and nothing is lost. For the same reason the highest value of Φ is reached for $\rho = 0$ because when assets are uncorrelated the manager can not exploit the information on the conventional assets investing only on the green ones.

Finally we note that the green bonus $\bar{\Delta}$ converges to $\bar{\Delta}_0$ as Φ goes to zero; Φ goes to

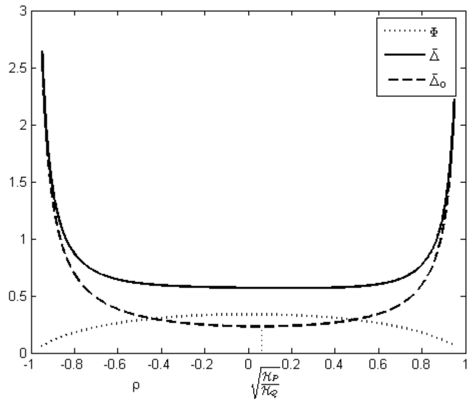

Figure 3: The green bonus $\bar{\Delta}$ (the solid line) is plotted against the correlation decomposed into $\bar{\Delta}_0$ (the dashed line) and Φ (the dotted line). The term $\bar{\Delta}_0$ goes to infinitive as $\rho \to \pm 1$ because when assets are so strictly correlated the diversification opportunity is so high that the manager would not give up it.

zero when the assets are strongly correlated and when $\frac{H}{H^g}$ approaches 1. The quantity $\frac{H}{H^g}$ goes to one when H^g goes to H that is when the manager is specialised in green.

To address the problem of selecting managers we require some extra assumptions; it is a common and meaningful assumption to consider the expertise as unknown. With a linear contract, if the expertise is not known and the principal wants to attract a manager with at least an expertise equal to a certain value $\hat{H}$, she will also attract any manager with lower expertise. However the investor would prefer to hire a manager with at least a suitable value of green expertise $\hat{H}^g$. We assume that the principal is able to acquire some information on the total expertise of the manager on the market, that is she knows H, but she ignores the manager's expertise on the green market H^g. The principal who decides to attract a manager of a known H level of expertise and at least a "green "expertise level of $\hat{H}^g$ must then set Φ to the value Φ_0 obtained by replacing $\hat{H}^g$ into (18). In this way the principal is sure to attract from the pool of manager of that total expertise those with green expertise greater than $\hat{H}^g$. See Figure 4. A suggestive interpretation of this result, following Kreps (1997), can be that the manager has something like an intrinsic motivation such that more motivated or dedicated green managers demand smaller extrinsic incentives.

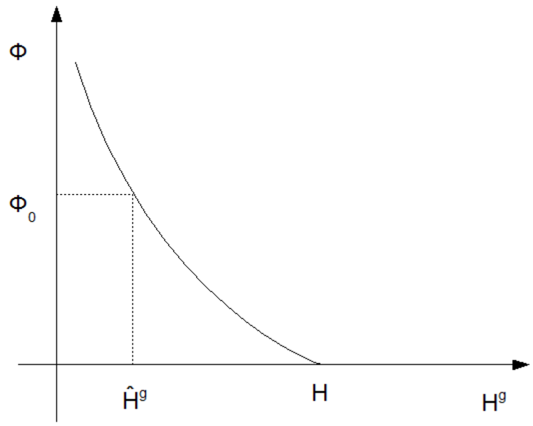

Figure 4: The principal chooses to attract a manager with at least a “green ”expertise level of $\hat{H}^g$, setting $\Phi = \Phi_0$ for a known H level of expertise, the principal is sure to attract from the pool of manager of that total expertise the ones with green expertise greater than $\hat{H}^g$.

5 Conclusion

This work is a first attempt to formulate a model of delegated portfolio management with constraints on the investment universe. In particular we focused on constraints given by SRI phylosophy and we studied the problem of setting a “green ”bonus and determined the minimum incentive required by a manager to give up earnings opportunity and diversification provided by the conventional assets. Such a quantity can be also interpreted as the cost of sustainability borne by the manager. Our approach also takes into account the possibility of extracting valuable information for example from typical Socially Responsible qualities, like Environmmental, Social or Governmental ones, as they may be considered as a way to improve the expertise of the portfolio manager. Our simple model permits to disentangle the effect on the green bonus of the statistical properties of market of the manager's skill and of his risk aversion. Finally we provide a definition for manager's expertise and we tackled the problem of selecting managers.

We leave for future researches the extension of the present approach to other kinds of contracts, for instance asymmetric and with limited liability, or where the bonus is related to a benchmark. It would also be interesting to consider a multi-period model to take into consideration issues connected to long and short-term incentives.

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